Equivalent Capacitance and ESR of Paralleled Capacitors

Parallel connection of capacitors is widely used in power electronics to decrease high frequency ripples and current stress, to decrease power dissipation and operating temperature, to shape frequency response, and to boost reliability. **Alexander Asinovski, Principal Engineer, Murata Power Solutions, Mansfield, USA**

Parallel connection of capacitors is widely used in power electronics to decrease high frequency ripples and current stress, to decrease power dissipation and operating temperature, to shape frequency response, and to boost reliability. Main questions a designer faces with regard to the parallel connection of capacitors are: What are equivalent capacitance and ESR (electric series resistance) values? What is high frequency ripple voltage? What are individual RMS currents?

If all capacitors in the parallel connection are identical with equal capacitance values, $C_{sk} = C$, $k = 1,2...N$ and equal ESR values $R_{sk} = R_s$ the answers are obvious: C_{se} is directly proportional to the number of capacitors N, $C_{se} = NC$, R_{se} is inversely proportional to N: $R_{se} = R_s / N$, ripple voltage V (RMS value) for a sinusoidal current excitation $i(t) = I\sqrt{2}\sin(2\pi ft)$ with frequency f, and RMS value I is

$$V = I\sqrt{R_{se}^2 + X_{se}^2}\,, \tag{1}$$

where $X_{se} = 1/(2\pi fC_{se})$ is the reactance of the equivalent capacitor C_{se} and individual RMS currents in the capacitors are identical: $I_k = I / N$.

In case capacitors in the parallel connection are not identical, with different capacitance C_{sk} and ESR R_{sk} values, the solution to the problem is not trivial. The direct approach would be obtaining an analytical expression for the input impedance of the parallel connection in the algebraic form $Z = \mathrm{Re}Z - j\,\mathrm{Im}\,Z$ and using the formulas $R_{se} = \mathrm{Re}Z$, $X_{se} = \mathrm{Im}\,Z$ and $C_{se} = 1/(2\pi fX_{se})$. A less complicated approach taken below is based on the conversion of series, C_{sk}, R_{sk} connections to equivalent parallel C_{pk}, R_{pk} connections. To obtain relationships between R_{pk} and R_{sk}, and also between C_{pk} and C_{sk}, set admittance Y_{pk} of the parallel C_{pk}, R_{pk} and admittance Y_{sk} of the series C_{sk}, R_{sk} connections equal to each other: $Y_{pk} = Y_{sk}$, $\mathrm{Re}(Y_{pk}) = \mathrm{RE}(Y_{sk})$ and $\mathrm{Im}(Y_{pk}) = \mathrm{Im}(Y_{sk})$. It follows:

$$C_{pk} = C_{sk} / \left[1 + \left(R_{sk} / X_{sk}\right)^2\right], \tag{2}$$

$$R_{pk} = \left(R_{sk}^2 + X_{sk}^2\right) / R_{sk}\,, \tag{3}$$

where

$$X_{sk} = 1/2(2\pi fc_{sk}) \tag{4}$$

is the reactance of the individual capacitor.

After individual parallel capacitance C_{pk} and resistance R_{pk} values are calculated according to (2) and (3), equivalent parallel capacitance C_{pe} can be easily found as the sum of C_{pk}

$$C_{pe} = \sum_{k=1}^{N} C_{pk} \tag{5}$$

and real part of equivalent admittance can be found as the sum of admittances $1/R_{pk}$. R_{pe} can be obtained as a reverse value of that sum:

$$R_{pe} = 1/\left[\sum_{k=1}^{N}\left(1/R_{pk}\right)\right]. \tag{6}$$

Equivalent series capacitance C_{se} and ESR R_{se} of the system can be found by conversion of the parallel C_{pe}, R_{pe} connection to the equivalent series connection C_{se}, R_{se}. To obtain relationships between C_{se} and C_{pe} and also between R_{se} and R_{pe}, set impedance Z_{pe} of the parallel C_{pe}, R_{pe} and impedance Z_{se} of the series C_{se}, R_{se} connections equal to each other: $Z_{pe} = Z_{se}$, $\mathrm{Re}\,Z_{pe} = \mathrm{Re}\,Z_{se}$ and $\mathrm{Im}\,Z_{pe} = \mathrm{Im}\,Z_{se}$. It follows:

$$C_{se} = C_{pe}\left[1 + \left(X_{pe} / R_{pe}\right)^2\right], \tag{7}$$

$$R_{se} = R_{pe} / \left[1 + \left(R_{pe} / X_{pe}\right)^2\right], \tag{8}$$

where

$$X_{pe} = 1/\left(2\pi fC_{pe}\right) \tag{9}$$

is reactance of the equivalent parallel capacitor C_{pe} (5).

Based on the analysis presented above, calculation procedure for equivalent series capacitance, ESR, voltage ripples , and RMS currents in the capacitors is as follows:

1. Calculate reactances of individual capacitances according to formula (4).
2. Determine equivalent parallel parameters C_{pk}, R_{pk} of the capacitors based on equations (2) and (3).
3. Calculate equivalent parallel capacitance C_{pe} of the structure, its reactance X_{pe}, and equivalent parallel resistance R_{pe} according to formulas (5), (9), and (6).
4. Calculate equivalent series capacitance C_{se} and ESR R_{se} of the structure according to formulas (7) and (8).
5. Obtain RMS ripple voltage V using equation (1).
6. Calculate RMS currents I_k in the capacitors based on the formula

$$I_k = V / \sqrt{R_{sk}^{\ 2} + X_{sk}^{\ 2}}\,. \tag{10}$$

It is worthwhile to note that ESR values are strong functions of frequency. A designer should use ESR data specified by capacitor manufacturers at a given frequency of operation. An example of a comprehensive source of data for ceramic and polymer aluminum electrolytic capacitors is found on the Murata Manufacturing Co., Ltd. (MMC) website http://ds.murata.co.jp/software/simsurfing/en-us/index.html.

To illustrate the calculation procedure let's determine equivalent parameters, voltage ripple, and current distribution for a parallel connection at of three ceramic capacitors GRM21BR60J226ME39L and one polymer capacitor ESASD40J107M015K00 from MMC.

Assuming the following input data: $f = 200$ kHz, $C_{s1} = C_{s2} = C_{s3} = 22\mu$F, $R_{s1} = R_{s2} = R_{s3} = 4m\Omega$ $C_{s4} = 100\mu$F, $R_{s4} = 8$mΩ, I = 2A.

1. For reactance of each individual capacitance according to formula (4) we have: $X_{s1} = X_{s2} = X_{s3} = 3.6\ m\Omega$, $X_{s4} = 0.8m\Omega$.
2. Equivalent parallel parameters C_{pk} , R_{pk} of the capacitors based on formulas (2) and (3) are: $C_{p1} = C_{p2} = C_{p3} = 21.7\ \mu F$, $R_{p1} = R_{p2} = R_{p3} = 331\ m\Omega$, $C_{p4} = 49.7\ \mu F$, $R_{p4} = 16m\Omega$.
3. For equivalent parallel capacitance C_{pe} , its reactance X_{pe} , and equivalent parallel resistance R_{pe} of the structure according to formulas (5), (9), and (6) we calculate: $C_{pe} = 115\mu F$, $X_{pe} = 6.9\ m\Omega$, $R_{pe} = 13.9\ m\Omega$.
4. Equivalent series capacitance C_{se} and ESR R_{se} according to formulas (7) and (8) are: $C_{se} = 143.4\mu F$, $R_{se} = 2.76m\Omega$.
5. For RMS ripple voltage based on equation (1) we obtain: $V = 12.4$ mV.
6. RMS currents according to formula (10) in ceramic and polymer capacitors are respectively: $I_1 = I_2 = I_3 = 341mA$, $I_4 = 1.1A$.